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# Dispersionless Surface Polaritons at Twist Boundaries of Crystals and in a Transition Layer between the Crystals

**A. N. Furs, V. M. Galynsky, and L. M. Barkovsky**

*Department of Theoretical Physics, Belarussian State University, Minsk, 220080 Belarus*

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**Abstract**—Dispersionless surface polaritons at twist boundaries of transparent uniaxial crystals are investigated. Exact analytical expressions for the central angles of the sectors of propagation of the surface polaritons are derived for arbitrary degrees of crystalline anisotropy $\eta = \varepsilon_{\parallel}/\varepsilon_{\perp} - 1$. The character of the change in the propagation sectors is determined with due regard for the isotropic transition layer between the crystals. © *2005 Pleiades Publishing, Inc.*

## INTRODUCTION

Dispersionless (singular) surface polaritons are the specific type of surface electromagnetic waves propagating along the interfaces between materials of different symmetries. These surface electromagnetic waves were theoretically predicted by Marchevskiĭ *et al.* [1] and D'yakonov [2] for the case of an interface between a positive uniaxial crystal and an isotropic medium. In contrast to surface polaritons at boundaries between strongly dispersive isotropic media with negative and positive permittivities [3], singular surface electromagnetic waves can be excited in weakly dispersive anisotropic, gyrotropic, and bianisotropic media [4, 5]. The possible directions of propagation of dispersionless surface polaritons form sectors in the plane of the interface. The arrangement and the angular width (i.e., the central angle) of these sectors are governed by the orientation of the crystallographic axes of the boundary materials with respect to the planes of the crystal cuts and by the degree of crystalline anisotropy. An important point is that the sectors with allowed directions of propagation of dispersionless surface polaritons can be dynamically modified in electro- and magneto-optical media by varying the external controlling electric and magnetic fields [6]. The inclusion of the anisotropy or bianisotropy of the materials substantially complicates the mathematical formalism used in the theory of surface polaritons. Therefore, the dispersion relations and the conditions of existence of surface electromagnetic waves can be conveniently established in the framework of the three-dimensional covariant formalism of surface impedance tensors and their integral representation [5, 7–9].

Optical anisotropy plays a crucial role, especially in the case of dispersionless surface excitations at a boundary formed by different cuts of the same crystal. Surface polaritons at twist boundaries of positive transparent uniaxial crystals were investigated by Averkiev and D'yakonov [10] and Darinskiĭ [11]. These boundaries are formed when a uniaxial crystal is cut by a plane passing through the optic axis and the halves of this crystal are rotated with respect to each other in such a way that the crystal axes appear to be crossed in the plane of the interface. In particular, Averkiev and D'yakonov [10] obtained dispersion relations for polaritons in the form of a system of algebraic equations and treated analytically the limiting cases of weak and strong anisotropies of a uniaxial crystal. Moreover, those authors derived approximate expressions describing the angular widths of the propagation sectors of surface polaritons in the aforementioned limiting cases. In this paper, the problem of dispersionless surface electromagnetic waves at twist boundaries of transparent uniaxial crystals is considered more comprehensively (as compared to the analyses performed in [10, 11]) within the formalism of surface impedance tensors. The exact analytical expressions characterizing the angular width of the propagation sectors of dispersionless surface polaritons are obtained using the direct tensor methods described in [12, 13]. These expressions are valid for any degree of anisotropy of positive uniaxial crystals $\eta = \varepsilon_{\parallel}/\varepsilon_{\perp} - 1$. Although the majority of natural crystals possess weak anisotropy in the visible spectral range, the use of modern technologies makes it possible to synthesize strongly anisotropic materials (such as composites, mesoscopic compounds, etc. [14–16]) for which the degree of anisotropy $\eta$ is of the order of 10.

In this paper, we also analyze how an isotropic transition layer affects the properties of dispersionless surface electromagnetic waves propagating along the interfaces in uniaxial crystals with crossed optic axes. The problem of transition layers and their influence on the reflection and refraction of light have been considered in a large number of publications (see [17, 18] and references therein). It is known that, for surface polaritons in resonant media with a negative permittivity, the inclusion of a transition layer leads to manifestation of a number of new effects, for example, splitting of dispersion curves [3]. However, the problem regarding the influence of a transition layer on the properties of dispersionless surface polaritons has not been adequately investigated. In this study, we will derive the dispersion relation for polaritons in an isotropic transition layer between crossed crystals and find its numerical solutions. It will be demonstrated that the presence of a transition layer between crystals brings about a change in the sector of possible propagation directions of surface electromagnetic waves and that, under certain conditions, no excitation of surface waves occurs in any one of the directions in the interface plane.

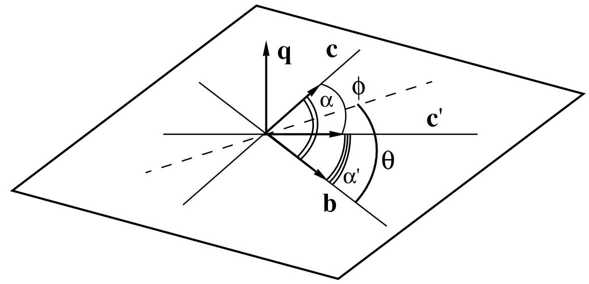


**Fig. 1.** Arrangement of the optic axes in the interface plane. For explanation, see text.

## SECTORS OF PROPAGATION OF DISPERSIONLESS SURFACE POLARITONS ALONG A TWIST BOUNDARY OF A UNIAXIAL CRYSTAL

Let us consider the propagation of surface electromagnetic waves along a planar interface formed by different cuts of the same nonmagnetic uniaxial crystal. It is assumed that, in this crystal, the **c** and **c**' optic axes lying on each side of the interface are parallel to it and make an angle $\phi$ ($0 \le \phi \le \pi/2$; see Fig. 1). The origin of the Cartesian coordinate system is located in the interface plane, and the $z$ axis is chosen parallel to the unit vector **q** normal to this plane. In this case, the crystal is characterized by the following inverse permittivity tensors:

in the range $z < 0$,

$$\varepsilon^{-1} = a + (b-a)\mathbf{c}\otimes\mathbf{c}, \quad \mathbf{c}^2 = 1;$$

in the range $z > 0$,

$$\varepsilon'^{-1} = a + (b-a)\mathbf{c}'\otimes\mathbf{c}', \quad \mathbf{c}'^2 = 1.$$

Here, $a = 1/\varepsilon_\perp$ and $b = 1/\varepsilon_\parallel$.

The dependences of the magnetic and electric field vectors of the surface electromagnetic waves on the radius vector **r** at the observation point and on the time $t$ in the half-space $z < 0$ are described by the expressions

$$\mathbf{H}(\mathbf{r}, t) = \sum_{s=1}^{2} C_s \mathbf{H}_s^0 \exp[ik(\mathbf{b} + \eta_s\mathbf{q})\mathbf{r} - i\omega t],$$

$$\mathbf{E}(\mathbf{r}, t) = \sum_{s=1}^{2} C_s \mathbf{E}_s^0 \exp[ik(\mathbf{b} + \eta_s\mathbf{q})\mathbf{r} - i\omega t],$$

where $\mathbf{H}_s^0$ and $\mathbf{E}_s^0$ are the amplitudes of partial waves at the interface, $C_s$ are the weighting factors, $\eta_s$ are the complex damping coefficients ($\mathrm{Im}\,\eta_s < 0$), **b** is the unit vector specifying the direction of wave propagation along the interface, and $k$ is the projection of the wave vector onto the direction of the unit vector **b**. In what follows, the quantities denoted by primes will refer to the crystal in the half-space $z > 0$. Hence, the magnetic and electric field vectors $\mathbf{H}'(\mathbf{r}, t)$ and $\mathbf{E}'(\mathbf{r}, t)$ at $z > 0$ are described by similar expressions, in which the quantities $\mathbf{H}_s^0$, $\mathbf{E}_s^0$, $C_s$, $\eta_s$, and $\mathbf{m}_s$ are replaced by the corresponding primed quantities and the condition for a decrease in the wave amplitude with an increase in the distance to the interface takes the form $\mathrm{Im}\,\eta_s' > 0$. The complex damping coefficients $\eta_s$ can be determined from the equation of normals $|\mathbf{m}^\times\varepsilon^{-1}(\omega)\mathbf{m}^\times + 1| = 0$ after substituting the complex-valued refraction vector **m** [12, 13] in the form $\mathbf{m}_s = ck(\mathbf{b} + \eta_s\mathbf{q})/\omega$ into this equation. In turn, the complex damping coefficients $\eta_s'$ can be obtained from a similar equation involving the tensor $\varepsilon'$.

The boundary conditions for the tangential components of the electric and magnetic fields can be written in the form

$$\mathbf{H}_\tau^0 = \sum_{s=1}^{2} C_s \mathbf{H}_{s\tau}^0 = \sum_{s=1}^{2} C_s' \mathbf{H}_{s\tau}'^0 = \mathbf{H}_\tau'^0,$$
$$[\mathbf{q}\mathbf{E}^0] = \sum_{s=1}^{2} C_s[\mathbf{q}\mathbf{E}_s^0] = \sum_{s=1}^{2} C_s'[\mathbf{q}\mathbf{E}_s'^0] = [\mathbf{q}\mathbf{E}'^0]. \tag{1}$$

Now, we introduce the surface impedance tensors $\gamma$ and $\gamma'$, which relate the tangential components of the electric and magnetic fields at the interface [19]:

$$[\mathbf{q}\mathbf{E}^0] = \gamma\mathbf{H}_\tau^0, \quad [\mathbf{q}\mathbf{E}'^0] = \gamma'\mathbf{H}_\tau'^0. \tag{2}$$

By eliminating the vectors $\mathbf{H}_\tau'^0$, $[\mathbf{q}\mathbf{E}^0]$, and $[\mathbf{q}\mathbf{E}'^0]$ from expressions (1) and (2), we obtain the equation

$$(\gamma - \gamma')\mathbf{H}_\tau^0 = 0. \tag{3}$$

Equation (3) has nonzero solutions $\mathbf{H}_\tau^0$ when the trace of the tensor adjoined to the tensor $\gamma - \gamma'$ is equal to zero [12, 13]; that is,

$$\overline{(\gamma - \gamma')}_t = 0. \tag{4}$$

Expression (4) is the dispersion relation for surface electromagnetic waves.

The form of the surface impedance tensors $\gamma$ and $\gamma'$ is determined both by the symmetry of the contacting crystals and by the mutual arrangement of the vectors **b**, **q**, and **a** = [**bq**] and the principal axes of the tensors $\varepsilon(\omega)$ and $\varepsilon'(\omega)$. For optically uniaxial crystals under the condition **qc** = **qc**' = 0, the tensors $\gamma$ and $\gamma'$ have the following form [20]:

$$\gamma = \frac{i}{\sqrt{a(d-\nu^2)}+\sqrt{b(a-\nu^2)}}$$
$$\times\left[\nu\left(d\sqrt{\frac{a-\nu^2}{d-\nu^2}}+\sqrt{ab}\right)\mathbf{b}\otimes\mathbf{b}\right.$$
$$+\nu(b-a)\sin\alpha\cos\alpha\sqrt{\frac{a-\nu^2}{d-\nu^2}}(\mathbf{b}\otimes\mathbf{a}+\mathbf{a}\otimes\mathbf{b}) \quad (5)$$
$$-\frac{1}{\nu}\left((a-\nu^2)\sqrt{ab}\right.$$
$$\left.\left.+[ab-\nu^2(a+b-d)]\sqrt{\frac{a-\nu^2}{d-\nu^2}}\right)\mathbf{a}\otimes\mathbf{a}\right],$$

$$\gamma' = -\frac{i}{\sqrt{a(d'-\nu^2)}+\sqrt{b(a-\nu^2)}}$$
$$\times\left[\nu\left(d'\sqrt{\frac{a-\nu^2}{d'-\nu^2}}+\sqrt{ab}\right)\mathbf{b}\otimes\mathbf{b}\right.$$
$$+\nu(b-a)\sin\alpha'\cos\alpha'\sqrt{\frac{a-\nu^2}{d'-\nu^2}}(\mathbf{b}\otimes\mathbf{a}+\mathbf{a}\otimes\mathbf{b}) \quad (6)$$
$$-\frac{1}{\nu}\left((a-\nu^2)\sqrt{ab}\right.$$
$$\left.\left.+[ab-\nu^2(a+b-d')]\sqrt{\frac{a-\nu^2}{d'-\nu^2}}\right)\mathbf{a}\otimes\mathbf{a}\right],$$

where

$$d = a\cos^2\alpha + b\sin^2\alpha, \quad d' = a\cos^2\alpha' + b\sin^2\alpha',$$

$\alpha$ is the angle between the vectors **b** and **c**, $\alpha' = \alpha - \phi$ is the angle between the vectors **b** and **c**' in the interface plane (Fig. 1), and $\nu = \omega/(ck)$ is the dimensionless frequency (the phase velocity of surface electromagnetic waves in terms of the velocity of light $c$ in free space). By substituting expressions (5) and (6) into relation (4), we obtain the dispersion relation

$$F(\nu) = 0, \quad (7)$$

$$F(\nu) = -\sqrt{ab}\sqrt{\frac{a-\nu^2}{d-\nu^2}} - \sqrt{ab}\sqrt{\frac{a-\nu^2}{d'-\nu^2}} - \frac{1}{[\sqrt{a(d-\nu^2)}+\sqrt{b(a-\nu^2)}][\sqrt{a(d'-\nu^2)}+\sqrt{b(a-\nu^2)}]}$$
$$\times\left\{\left(d\sqrt{\frac{a-\nu^2}{d-\nu^2}}+\sqrt{ab}\right)\left[(a-\nu^2)\sqrt{ab}+[ab-\nu^2(a+b-d')]\sqrt{\frac{a-\nu^2}{d'-\nu^2}}\right]\right.$$
$$+\left(d'\sqrt{\frac{a-\nu^2}{d'-\nu^2}}+\sqrt{ab}\right)\left[(a-\nu^2)\sqrt{ab}+[ab-\nu^2(a+b-d)]\sqrt{\frac{a-\nu^2}{d-\nu^2}}\right] \quad (8)$$
$$\left.+2\nu^2(a-b)^2\sqrt{\frac{a-\nu^2}{d-\nu^2}}\sqrt{\frac{a-\nu^2}{d'-\nu^2}}\sin\alpha\cos\alpha\sin\alpha'\cos\alpha'\right\}.$$

For the specified parameters $a$, $b$, $\phi$, and $\alpha$, the solution $\nu = \nu_S$ of dispersion relation (7) describes the surface wave propagating along the vector **b** only in the case where the damping coefficients $\eta_s$ and $\eta'_s$ are complex quantities, i.e., where the energy of the electromagnetic field of the surface wave is localized in the vicinity of the interface in both crystals. This means that the dimensionless phase velocity of the surface wave should be less than the limiting value $\nu_L$; i.e., it should satisfy the condition [8]

$$0 \le \nu_S < \nu_L.$$

In this case, we have $\nu_L = \sqrt{a}$ for negative crystals ($a < b$) and $\nu_L = \min(\sqrt{d}, \sqrt{d'})$ for positive crystals ($a > b$). If dispersion relation (7) does not have solutions on the interval $[0, \nu_L)$ (the sublight interval [8]), the surface wave cannot propagate along the vector **b**.

It can be shown that the function $F(\nu)$ defined by expression (8) is a monotonic function of the frequency. This function is negative at $\nu = 0$ and equal to zero at $\nu = \nu_L = \sqrt{a}$ for negative crystals. Consequently, surface excitations are impossible at the interface formed by different cuts of a negative crystal. Thus, hereinafter, we will consider waves at the interface of positive crystals ($a > b$).

If a surface wave can propagate in the direction **b**, it can also propagate in the opposite direction –**b** because the function $F(\nu)$ remains unchanged upon the replacements $\alpha \longrightarrow \pi + \alpha$ and $\alpha' \longrightarrow \pi + \alpha'$. Moreover, this function is invariant with respect to the changes $\alpha \longrightarrow -\alpha'$ and $\alpha' \longrightarrow -\alpha$. Therefore, the set of directions **b** in which the surface waves can propagate is reflection-symmetric in the interface plane upon reflections with respect to the bisectrix of the angle formed by the vectors **c** and **c**' and upon reflections with respect to the perpendicular to this bisectrix. Consequently, it is sufficient to take into account only the directions **b** specified by the angles $\alpha \in [\phi/2, \pi/2 + \phi/2]$. In this case, we have $d < d'$ and $\nu_L = \sqrt{d}$.

It should be noted that the function $F(\nu)$ entering into dispersion relation (7) is a complicated function of the frequency $\nu$. Nonetheless, it is an easy matter to determine the region of existence of solutions to this dispersion relation. Since the function $F(\nu)$ is monotonic and negative at $\nu = 0$, the region of existence of these solutions can be determined from the condition

$$\lim_{\nu \to \nu_L = \sqrt{d}} F(\nu) > 0. \tag{9}$$

In the function $F(\nu)$, it is possible to separate the coefficient of the component that diverges as $(d - \nu^2)^{-1/2}$. By imposing the constraint that this coefficient must be positive, we obtain the inequality

$$\begin{aligned} &-2b\sqrt{ab}\sin\alpha\sqrt{\sin^2\alpha - \sin^2\alpha'} \\ &- 2ab(\sin^2\alpha - \sin^2\alpha') \\ &+ d(a-b)(\sin^2\alpha - \sin^2\alpha')^2 \\ &+ d(a-b)(\sin\alpha\cos\alpha - \sin\alpha'\cos\alpha')^2 > 0. \end{aligned}$$

Then, it is convenient to introduce the angle $\theta$ between the vector **b**, which specifies the direction of wave propagation along the interface, and the bisectrix of the angle made by the vectors **c** and **c**' (Fig. 1). As a result, we have the equalities $\alpha = \theta + \phi/2$ and $\alpha' = \theta - \phi/2$ and condition (9) takes the form

$$A(x) > 0, \tag{10}$$

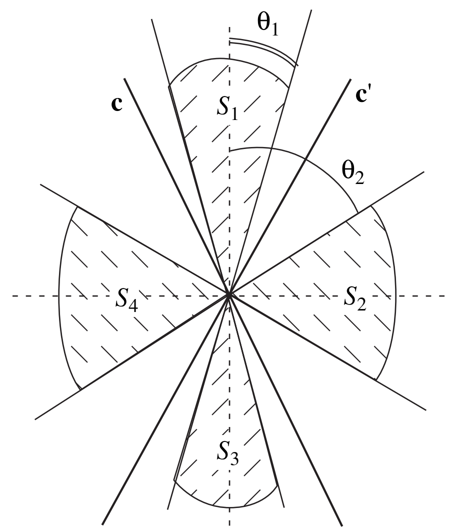


**Fig. 2.** Sectors of the allowed propagation directions **b** of surface polaritons.

where

$$\begin{aligned} A(x) = {} & \eta\left(1 + \eta\sin^2\frac{\phi}{2}\right)\left(\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)^{3/2} x^2 \\ & - \sqrt{1+\eta}\cos\frac{\phi}{2} x^{3/2} \\ & - 2\left(1 + \eta\sin^2\frac{\phi}{2}\right)\left(1 + \eta\cos^2\frac{\phi}{2}\right)\sqrt{\sin\frac{\phi}{2}\cos\frac{\phi}{2}}\, x \\ & - \sqrt{1+\eta}\sin\frac{\phi}{2} x^{1/2} + \eta\left(1 + \eta\cos^2\frac{\phi}{2}\right)\left(\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)^{3/2}. \end{aligned} \tag{11}$$

Here, $x = \tan\theta$ and $\eta = a/b - 1 = \varepsilon_{\parallel}(\omega)/\varepsilon_{\perp}(\omega) - 1$ is the degree of anisotropy of the uniaxial crystal. Therefore, condition (10) determines the angles $\theta$ corresponding to the directions **b** in which surface electromagnetic waves can propagate. In the interface plane, these directions form four sectors, namely, $S_1$, $S_2$, $S_3$, and $S_4$ (Fig. 2). The positions of the sectors are characterized by the angles $\theta_1 = \arctan x_1$ and $\theta_2 = \arctan x_2$ ($0 < \theta_1 < \theta_2 < \pi/2$), where $x_1$ and $x_2$ are the positive roots of the equation $A(x) = 0$. It is evident that the angular width is equal to $2\theta_1$ for the sectors $S_1$ and $S_3$ and $\pi - 2\theta_2$ for the sectors $S_2$ and $S_4$.

Figure 3 shows the dependences of the critical angles $\theta_1$ and $\theta_2$ on the degree of anisotropy $\eta$ at constant angles $\phi$ of crossing of the axes. As can be seen from Fig. 3, the higher the degree of anisotropy of the crystal, the larger the angular width of the sectors $S_1, \ldots, S_4$. This situation is typical of dispersionless surface electromagnetic waves at an interface of anisotropic materials.

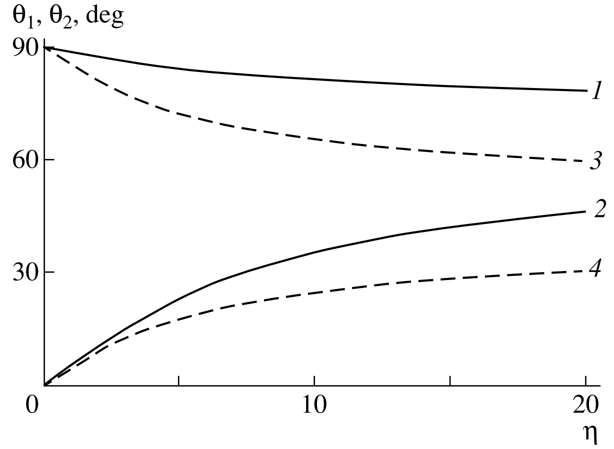


**Fig. 3.** Dependences of the critical angles $\theta_1$ and $\theta_2$ ($\theta_2 > \theta_1$) on the degree of anisotropy $\eta$ for angles $\phi$ = (*1*, *2*) $\pi/4$ and (*3*, *4*) $\pi/2$.

For crystals with weak and strong anisotropies, simple approximate expressions for the critical angles $\theta_1$ and $\theta_2$ can be derived from the equation $A(x) = 0$. In particular, for a weakly anisotropic material ($\eta \ll 1$), the critical angle $\theta_1$ is small and, hence, we have $x = \tan\theta_1 \approx \theta_1$. Then, we introduce the variable $y$ in such a way that $x = \eta^2 y$. By retaining only the terms linear in the degree of anisotropy $\eta$ in expression (11) for the quantity $A(x)$, we find that $y = \sin(\phi/2)\cos^3(\phi/2)$. As a result, the angle $\theta_1$ can be determined from the approximate formula

$$\theta_1 \approx \eta^2 \sin\frac{\phi}{2}\cos^3\frac{\phi}{2}. \tag{12}$$

Similarly, the angle $\theta_2$ can be obtained from the approximate expression

$$\theta_2 \approx \frac{\pi}{2} - \eta^2 \sin^3\frac{\phi}{2}\cos\frac{\phi}{2}. \tag{13}$$

The angular width of the sectors $S_1, \ldots, S_4$ turns out to be proportional to $\eta^2$. For strongly anisotropic materials ($\eta \gg 1$), we have

$$\theta_{1,2} = \frac{\pi}{2} - \frac{\phi}{2} \pm \frac{1}{\sqrt{\eta}}. \tag{14}$$

It should be emphasized that approximate expressions (12)–(14) were derived for the first time by Averkiev and D'yakonov [10].

If the direction **b** either coincides with the bisectrix of the angle formed by the vectors **c** and **c**' or is perpendicular to this bisectrix, the surface wave can propagate at any crossing angles $\phi$ ($0 < \phi \le \pi/2$) and its polarization is linear [11]. Actually, in this case, we have $\theta = n\pi/2$ ($n = 0, 1, 2, 3$) and inequality (10) holds at any parameters $a$ and $b$ satisfying the condition $a > b$.

Now, we estimate the error in approximate expressions (12) and (13) and determine the range of their applicability. Without a loss in generality, the error can be estimated for the angle $\theta_1$ under the assumption that the crossing angle $\phi$ varies from 0 to $\pi$. It is also assumed that $x_1 \approx \theta_1$ (defined by expression (12)) and that $x_1$ and $\tilde{x}$ are the approximate and exact solutions of the equation $A(x) = 0$, respectively. The function $A(\tilde{x})$ can be expanded into a Taylor series in the vicinity of the point $x = x_1$:

$$A(\tilde{x}) = 0 = A(x_1) + \left.\frac{dA}{dx}\right|_{x = x_1}(\tilde{x} - x_1) + \ldots .$$

Then, the absolute ($\Delta$) and relative ($\delta$) errors in the calculation of the angle $\theta_1$ from expression (12) can be written in the form

$$\Delta = \tilde{x} - x_1 = -A(x_1)/\left.\frac{dA}{dx}\right|_{x = x_1},$$

$$\delta = \frac{\tilde{x} - x_1}{x_1} = -\frac{1}{x_1}A(x_1)/\left.\frac{dA}{dx}\right|_{x = x_1}.$$

Taking into account expression (11), we obtain

$$A(x_1) = -\eta^2\left(\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)^{3/2}\left(\frac{1}{2} + \cos^2\frac{\phi}{2}\right) + O(\eta^3),$$

$$\left.\frac{dA}{dx}\right|_{x = x_1} = -\frac{\sqrt{\sin\phi/2}}{2\eta\cos^{3/2}\phi} + O(1).$$

As a result, the relative error appears to be proportional to the degree of anisotropy $\eta$, that is,

$$\delta = -\eta(1 + 2\cos^2\phi/2),$$

and reaches the maximum absolute value $-3\eta$ at the crossing angle $\phi = 0$. If the magnitude $|\delta|$ is limited by 0.1, we can make the inference that the approximate expressions (12) and (13) can be used for degrees of anisotropy in the range $0 \le \eta < 0.03$, whereas the critical angles $\theta_1$ and $\theta_2$ and the angular widths of the sectors $S_1, \ldots, S_4$ at larger values of $\eta$ should be calculated from exact expressions (10) and (11).

## DISPERSIONLESS POLARITONS IN AN ISOTROPIC TRANSITION LAYER BETWEEN CROSSED UNIAXIAL CRYSTALS

The dispersion relation given by formula (7) was derived under the assumption that the cuts of the uniaxial crystal form a sharp boundary. In actual fact, the dielectric properties of the surface region of the crystal differ from those of the crystal regions located at a considerable distance from the interface. This difference brings about the formation of a transition layer in the vicinity of the interface. The existence of the transition layer between the crystals leads to a change in the surface excitation spectrum. This provides additional information on the physical properties of surface regions in the crystals [3].

Let us analyze how a transition layer affects the properties of dispersionless surface polaritons at twist boundaries of positive uniaxial crystals. It is assumed that the transition layer of thickness $l$ is isotropic, has a permittivity $\varepsilon_s$, and occupies the region $0 < z < l$. The uniaxial crystals with unit vectors $\mathbf{c}$ and $\mathbf{c}'$ of the optic axes aligned parallel to the interfaces are located in the regions $z < 0$ and $z > l$, respectively.

In order to derive the dispersion relation, we will use the expression relating the tangential components of the magnetic and electric fields at the interfaces $z = 0$ and $z = l$ through the characteristic matrix (propagator) $\mathbb{P}$ of the layer; that is,

$$\begin{pmatrix} \mathbf{H}_\tau'^0 \\ [\mathbf{q}\mathbf{E}'^0] \end{pmatrix} = \mathbb{P}\begin{pmatrix} \mathbf{H}_\tau^0 \\ [\mathbf{q}\mathbf{E}^0] \end{pmatrix}, \quad \mathbb{P} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}. \quad (15)$$

The elements $P_{11}$, $P_{12}$, $P_{21}$, and $P_{22}$ of the $6 \times 6$ matrix $\mathbb{P}$ are the planar tensors in the three-dimensional space. For an isotropic transition layer, these matrix elements can be written in the form [20]

$$P_{11} = P_{22} = \cosh\left(\frac{2\pi L}{\nu}\sqrt{1-\nu^2\varepsilon_s}\right)I,$$

$$P_{12} = i\sinh\left(\frac{2\pi L}{\nu}\sqrt{1-\nu^2\varepsilon_s}\right)$$

$$\times\left(\frac{\varepsilon_s\nu}{\sqrt{1-\nu^2\varepsilon_s}}\mathbf{a}\otimes\mathbf{a} - \frac{\sqrt{1-\nu^2\varepsilon_s}}{\nu}\mathbf{b}\otimes\mathbf{b}\right), \quad (16)$$

$$P_{21} = i\sinh\left(\frac{2\pi L}{\nu}\sqrt{1-\nu^2\varepsilon_s}\right)$$

$$\times\left(-\frac{\sqrt{1-\nu^2\varepsilon_s}}{\varepsilon_s\nu}\mathbf{a}\otimes\mathbf{a} + \frac{\nu}{\sqrt{1-\nu^2\varepsilon_s}}\mathbf{b}\otimes\mathbf{b}\right),$$

where $L = l/\lambda = \nu k l/2\pi$ is the layer thickness in terms of wavelengths in free space and $I = 1 - \mathbf{q}\otimes\mathbf{q} = \mathbf{b}\otimes\mathbf{b} + \mathbf{a}\otimes\mathbf{a}$ is the projective operator on the interface plane. By multiplying both sides of equality (15) from the left by the $3 \times 6$ matrix $(\gamma' - I)$ and taking into account the second formula (2) for the primed quantities, we derive the following relation between the tangential components of the electric and magnetic fields at the interface $z = 0$:

$$[\mathbf{q}\mathbf{E}^0] = (\gamma' P_{12} - P_{22})^{-}(P_{21} - \gamma' P_{11})\mathbf{H}_\tau^0, \quad (17)$$

where the sign $^{-}$ indicates the tensor pseudoinversion (the inversion in the two-dimensional space that is orthogonal to the vector $\mathbf{q}$). In expression (17), the tensor coefficient

$$\gamma_{\text{eff}} = (\gamma' P_{12} - P_{22})^{-}(P_{21} - \gamma' P_{11}) \quad (18)$$

is the effective tensor of the surface impedances of the crystal at $z > l$ and the transition layer. Taking into account expression (17) and the first formula (2), we obtain the following dispersion relation for surface electromagnetic waves localized at the interface $z = 0$:

$$(\overline{\gamma_{\text{eff}} - \gamma})_t = 0. \quad (19)$$

In dispersion relation (19), as compared to dispersion relation (4), the surface impedance tensor $\gamma'$ is replaced by the effective tensor $\gamma_{\text{eff}}$. The dispersion relation in the explicit form can be obtained by substituting expressions (18), (16), (5), and (6) into formula (19); however, it has a cumbersome form and is not presented here to save space.

In the limiting case $L = 0$ (i.e., the transition layer is absent), the matrix $\mathbb{P}$ has the elements $P_{11} = P_{22} = I$ and $P_{12} = P_{21} = 0$, the surface impedance tensors $\gamma_{\text{eff}}$ and $\gamma'$ coincide with each other, and dispersion relation (18) is transformed into dispersion relation (4).

It can be demonstrated that, in the limiting case $L \longrightarrow \infty$, the effective tensor $\gamma_{\text{eff}}$ becomes equal to the surface impedance tensor of the isotropic medium with the permittivity $\varepsilon_s$; that is,

$$\gamma_s = -\frac{i\nu}{\sqrt{1-\varepsilon_s\nu^2}}\mathbf{b}\otimes\mathbf{b} + \frac{i\sqrt{1-\varepsilon_s\nu^2}}{\varepsilon_s\nu}\mathbf{a}\otimes\mathbf{a}.$$

This means that, when analyzing the excitations of surface waves at the interface $z = 0$, we can ignore the fields at the other interface. Therefore, we change over to the problem of the propagation of dispersionless surface electromagnetic waves along an interface between a uniaxial crystal and an isotropic medium. This problem was considered by D'yakonov [2] (see also [8, 9]). In this case, the necessary condition of existence of surface waves can be written in the form [2]

$$\varepsilon_\parallel > \varepsilon_s > \varepsilon_\perp \quad (20)$$

and the allowed directions of propagation of surface waves are determined by the angles $\alpha$ lying in the intervals $(\alpha_{\min}, \alpha_{\max})$, $(\pi - \alpha_{\max}, \pi - \alpha_{\min})$, $(\pi + \alpha_{\min}, \pi + \alpha_{\max})$, and $(2\pi - \alpha_{\max}, 2\pi - \alpha_{\min})$. Here, the angles $\alpha_{\min}$ and $\alpha_{\max}$ are given by the expressions

$$\sin^2\alpha_{\min} = \frac{\xi}{2}\{1 - \eta\xi + [(1-\eta\xi)^2 + 4\eta]^{1/2}\},$$

$$\sin^2\alpha_{\max} = \frac{(1+\eta)^3\xi}{(1+\eta)^2(1+\eta\xi) - \eta^2(1-\xi)^2},$$

and $\xi = (\varepsilon_s - \varepsilon_\perp)/(\varepsilon_\parallel - \varepsilon_\perp)$. The sectors in the interface plane corresponding to these intervals are designated as $Q_1$, $Q_2$, $Q_3$, and $Q_4$, respectively.

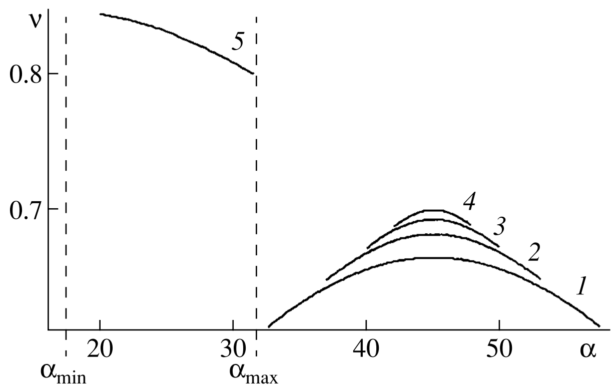


**Fig. 4.** Solutions of dispersion relation (20) for the nonoverlapping sectors $S_1$ ($S_2$) and $Q_1$ ($a = 0.8$, $b = 0.2$, $a' = 0.71$) for layer thicknesses $L$ = (*1*) 0, (*2*) 0.05, (*3*) 0.10, (*4*) 0.15, and (*5*) 14.00.

It is evident that a continuous change in the layer thickness $L$ from zero to infinity should be accompanied by the transformation of the sectors $S_1, \ldots, S_4$ (Fig. 2) into the sectors $Q_1, \ldots, Q_4$. We found the numerical solutions to dispersion relation (19) for different layer thicknesses $L$ and established the qualitative character of this transformation. By assuming that condition (20) is satisfied, we considered the following situations: (1) the sectors $S_1$ (or $S_2$) and $Q_1$ do not overlap for the specified orientation of the **c** and **c**' axes of the crystals, (2) the sector $Q_1$ lies inside the sector $S_1$ (or $S_2$), and (3) the sectors $Q_1$ and $S_1$ (or $S_2$) partially overlap.

In the first case, an increase in the layer thickness $L$ leads to a decrease in the angular width of the sectors $S_1, \ldots, S_4$ and, at a layer thickness $L_1$, these sectors disappear altogether. In the layer thickness interval ($L_1$, $L_2$), the dispersion relation has no solutions and surface excitations are impossible (Fig. 4). With a further increase in the layer thickness $L$ ($L > L_2$), the sectors of directions in which surface waves can propagate again appear in the interface plane. The angular width of these sectors increases, and, in the limit $L \longrightarrow \infty$, they transform into the sectors $Q_1, \ldots, Q_4$. It should be noted that, at a layer thickness $L > L_2$, the electromagnetic field amplitude on each side of the interface $z = 0$ decreases exponentially with an increase in $|z|$ but the wave in the uniaxial crystal ($z > l$) is a body wave.

In the second case, the dispersion relation has solutions at any thicknesses $L$ of the isotropic transition layer. For example, the angle $\phi$ of crossing of the axes can be chosen in such a way as to satisfy the equality

$$\phi = \alpha_{min} + \alpha_{max}.$$

Then, the midline of the sector $Q_1$ coincides with the bisectrix of the angle formed by the **c** and **c**' optic axes. As the layer thickness $L$ increases, the angular width of the sectors of allowed propagation directions initially decreases and then increases. As a result, the sectors $S_1, \ldots, S_4$ continuously transform into the sectors $Q_1, \ldots, Q_4$.

In the third case, the transformation of the sectors $S_1, \ldots, S_4$ into the sectors $Q_1, \ldots, Q_4$ occurs in a manner similar to that observed either in the first case or in the second case depending on the degree of overlapping of the sectors.

Finally, the permittivity $\varepsilon_s$ of the transition layer can be chosen in such a way that condition (20) will not be satisfied. In this case, an increase in the layer thickness $L$ results in a decrease in the angular width of the sectors $S_1, \ldots, S_4$ and the dispersion relation has no solutions for layer thicknesses exceeding a limiting value.

## CONCLUSIONS

Thus, the above analysis of dispersion relation (7) for surface polaritons propagating along a sharp interface formed by different cuts of the same positive uniaxial crystal has demonstrated that the possible directions of propagation of the dispersionless surface polaritons lie in sectors whose midlines coincide either with the bisectrix of the angle between the **c** and **c**' optic axes or with the perpendicular to this bisectrix. Exact analytical expressions for the angular widths of these sectors are derived for arbitrary degrees of crystalline anisotropy $\eta = \varepsilon_{\parallel}/\varepsilon_{\perp} - 1$. For weakly and strongly anisotropic crystals, these expressions coincide with the approximate formulas obtained earlier in [10].

A dispersion relation is derived for surface electromagnetic waves propagating in a "uniaxial crystal–isotropic transition layer–uniaxial crystal" structure in which the optic axes of the crystals lie in the interface planes. Numerical solutions to this dispersion relation are obtained. It has been established that, if the sectors $S_1, \ldots, S_4$ do not overlap with the sectors $Q_1, \ldots, Q_4$, no excitation of surface polaritons occurs in any one of the directions in the interface planes in structures with transition layers whose thicknesses are of the order of several wavelengths.

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